Centre Number						Candidate Number				
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Other Names										
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MS03

Unit Statistics 3

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions in the spaces provided.

1 A consumer report claimed that more than 25 per cent of visitors to a theme park were dissatisfied with the catering facilities provided.

In a survey, 375 visitors who had used the catering facilities were interviewed independently, and 108 of them stated that they were dissatisfied with the catering facilities provided.

(a) Test, at the 2% level of significance, the consumer report's claim. *(6 marks)*

(b) State an assumption about the 375 visitors that was necessary in order for the hypothesis test in part **(a)** to be valid. *(1 mark)*

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2 The number of emergency calls received by a fire station may be modelled by a Poisson distribution.

During a given period of 13 weeks, the station received a total of 108 emergency calls.

(a) Construct an approximate 98% confidence interval for the average **weekly** number of emergency calls received by the station. *(5 marks)*

(b) Hence comment on the station officer's claim that the station receives an average of one emergency call **per day**. *(2 marks)*

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3 An IT help desk has three telephone stations: Alpha, Beta and Gamma. Each of these stations deals only with telephone enquiries.

The probability that an enquiry is received at Alpha is 0.60.
The probability that an enquiry is received at Beta is 0.25.
The probability that an enquiry is received at Gamma is 0.15.

Each enquiry is resolved at the station that receives the enquiry. The **percentages** of enquiries resolved within various times at **each** station are shown in the table.

		Time		
		$\leqslant$ 1 hour	**$\leqslant$ 24 hours**	**$\leqslant$ 72 hours**
Station	**Alpha**	55	80	100
	Beta	60	85	100
	Gamma	40	75	100

For example:
80 per cent of enquiries received at Alpha are resolved within 24 hours;
25 per cent of enquiries received at Alpha take between 1 hour and 24 hours to resolve.

(a) Find the probability that an enquiry, selected at random, is:

(i) resolved at Gamma; *(1 mark)*

(ii) resolved at Alpha within 1 hour; *(1 mark)*

(iii) resolved within 24 hours; *(2 marks)*

(iv) received at Beta, given that it is resolved within 24 hours. *(3 marks)*

(b) A random sample of 3 enquiries was selected.

Given that all 3 enquiries were resolved within 24 hours, calculate the probability that they were all received at:

(i) Beta; *(2 marks)*

(ii) the same station. *(4 marks)*

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4 The waiting time at a hospital's A&E department may be modelled by a normal distribution with mean μ and standard deviation $\frac{\mu}{2}$.

The department's manager wishes a 95% confidence interval for μ to be constructed such that it has a width of at most $0.2\,\mu$.

Calculate, to the nearest 10, an estimate of the minimum sample size necessary in order to achieve the manager's wish. *(5 marks)*

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5 An examination of 160 e-mails received by Gopal showed that 72 had attachments.

An examination of 250 e-mails received by Haley showed that 102 had attachments.

Stating **two** necessary assumptions about the selection of e-mails, construct an approximate 99% confidence interval for the difference between the proportion of e-mails received by Gopal that have attachments and the proportion of e-mails received by Haley that have attachments. *(8 marks)*

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6 The weight, X grams, of a dressed pheasant may be modelled by a normal random variable with a mean of 1000 and a standard deviation of 120.

Pairs of dressed pheasants are selected for packing into boxes. The total weight of a pair, $Y = X_1 + X_2$ grams, may be modelled by a normal distribution with a mean of 2000 and a standard deviation of 140.

(a) (i) Show that $\text{Cov}(X_1, X_2) = -4600$. *(3 marks)*

(ii) Given that $X_1 - X_2$ may be assumed to be normally distributed, determine the probability that the difference between the weights of a selected pair of dressed pheasants exceeds 250 grams. *(5 marks)*

(b) The weight of a box is independent of the total weight of a pair of dressed pheasants, and is normally distributed with a mean of 500 grams and a standard deviation of 40 grams.

Determine the probability that a box containing a pair of dressed pheasants weighs less than 2750 grams. *(5 marks)*

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7 (a) The random variable X has a Poisson distribution with $\mathrm{E}(X) = \lambda$.

(i) Prove, from first principles, that $\mathrm{E}(X(X-1)) = \lambda^2$. *(3 marks)*

(ii) Hence deduce that $\mathrm{Var}(X) = \mathrm{E}(X)$. *(2 marks)*

(b) The random variable Y has a Poisson distribution with $\mathrm{E}(Y) = 2.5$.

Given that $Z = 4Y + 30$:

(i) show that $\mathrm{Var}(Z) = \mathrm{E}(Z)$; *(3 marks)*

(ii) give a reason why the distribution of Z is **not** Poisson. *(1 mark)*

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8 The tensile strength of rope is measured in kilograms.

The standard deviation of the tensile strength of a particular design of 10 mm diameter rope is known to be 285 kilograms. A retail organisation, which buys such rope from two manufacturers, A and B, wishes to compare their ropes for mean tensile strength.

The mean tensile strength, $\bar{x}$, of a random sample of 80 lengths from manufacturer A was 3770 kilograms.

The mean tensile strength, $\bar{y}$, of a random sample of 120 lengths from manufacturer B was 3695 kilograms.

(a) (i) Test, at the 5% level of significance, the hypothesis that there is no difference between the mean tensile strength of rope from manufacturer A and that of rope from manufacturer B. *(6 marks)*

(ii) Why was it **not** necessary to know the distributions of tensile strength in order for your test in part **(a)(i)** to be valid? *(1 mark)*

(b) (i) Deduce that, for your test in part **(a)(i)**, the critical values of $(\bar{x} - \bar{y})$ are ± 80.63, correct to two decimal places. *(2 marks)*

(ii) In fact, the mean tensile strength of rope from manufacturer A exceeds that of rope from manufacturer B by 125 kilograms.

Determine the probability of a Type II error for a test of the hypothesis in part **(a)(i)** at the 5% level of significance, based upon a random sample of 80 lengths from manufacturer A and a random sample of 120 lengths from manufacturer B. *(4 marks)*

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END OF QUESTIONS

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